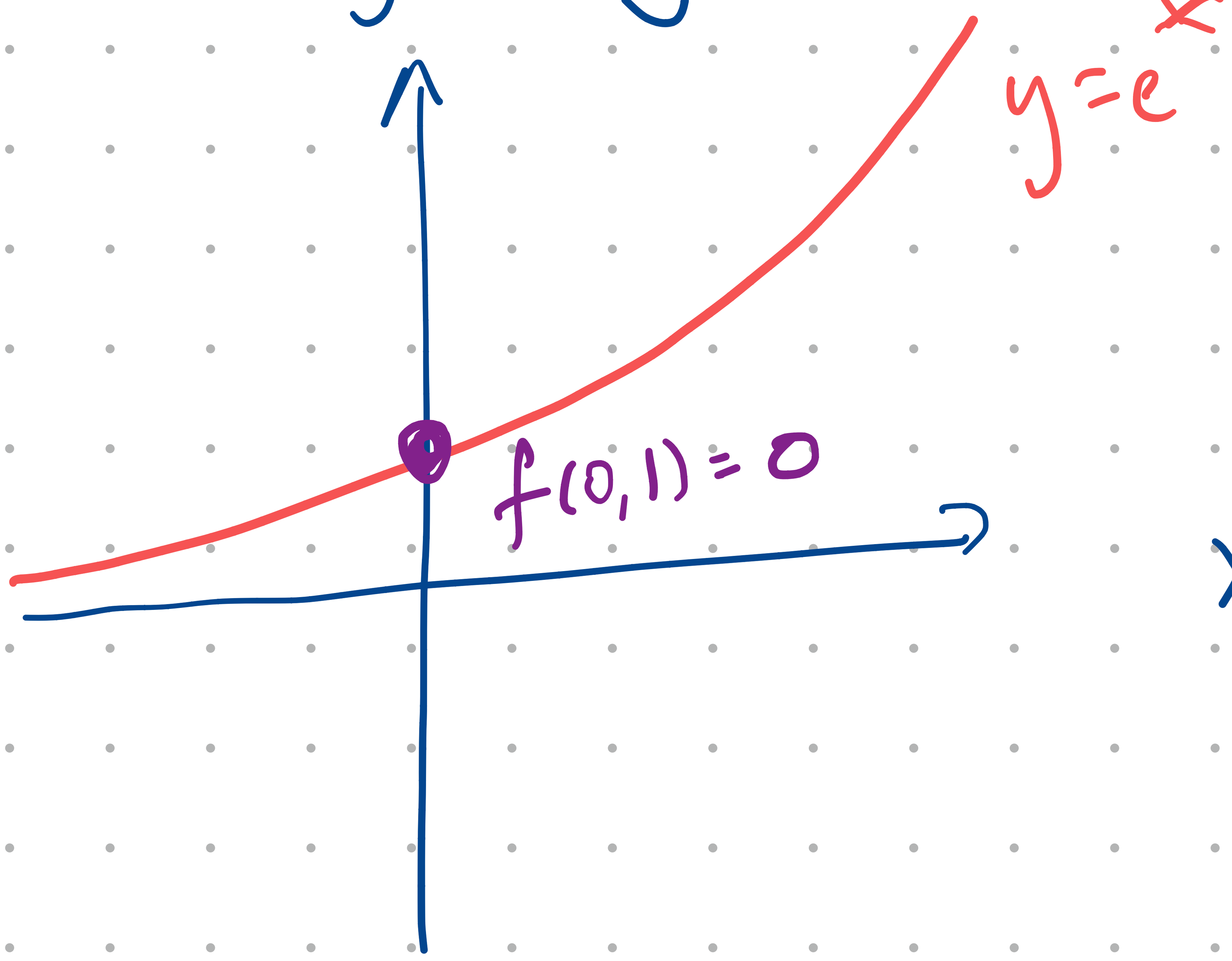
Quiz 5

① $f(x,y) = x^2 y$ on constraint $y = e^x$



in other words, does $x^2 e^x$ have an abs max/min?

No abs max, but $x^2 e^x \geq 0$

and $0^2 e^0 = 0$, so

$f(0,1) = 0$ is absolute min.

(b/c $f(x,y) \geq f(0,1)$ for all x, y on $y = e^x$)

(2) $f(x,y) =$ some scary expression

constraint: $x^2 + y^6 \leq 7$

Just need to observe

- $f(x,y)$ is continuous

- region is closed (b/c didn't use $<$ or $>$ to define it)

- region is bounded

  (Note: $x^2 + y^3 \leq 7$ is <u>not</u> bounded.)

b/c from $x^2 + y^6 \leq 7$ already know

$-\sqrt{7} \leq x \leq \sqrt{7}$ & $-7^{1/6} \leq y \leq 7^{1/6}$

So can invoke EVT to deduce existence of max and min.

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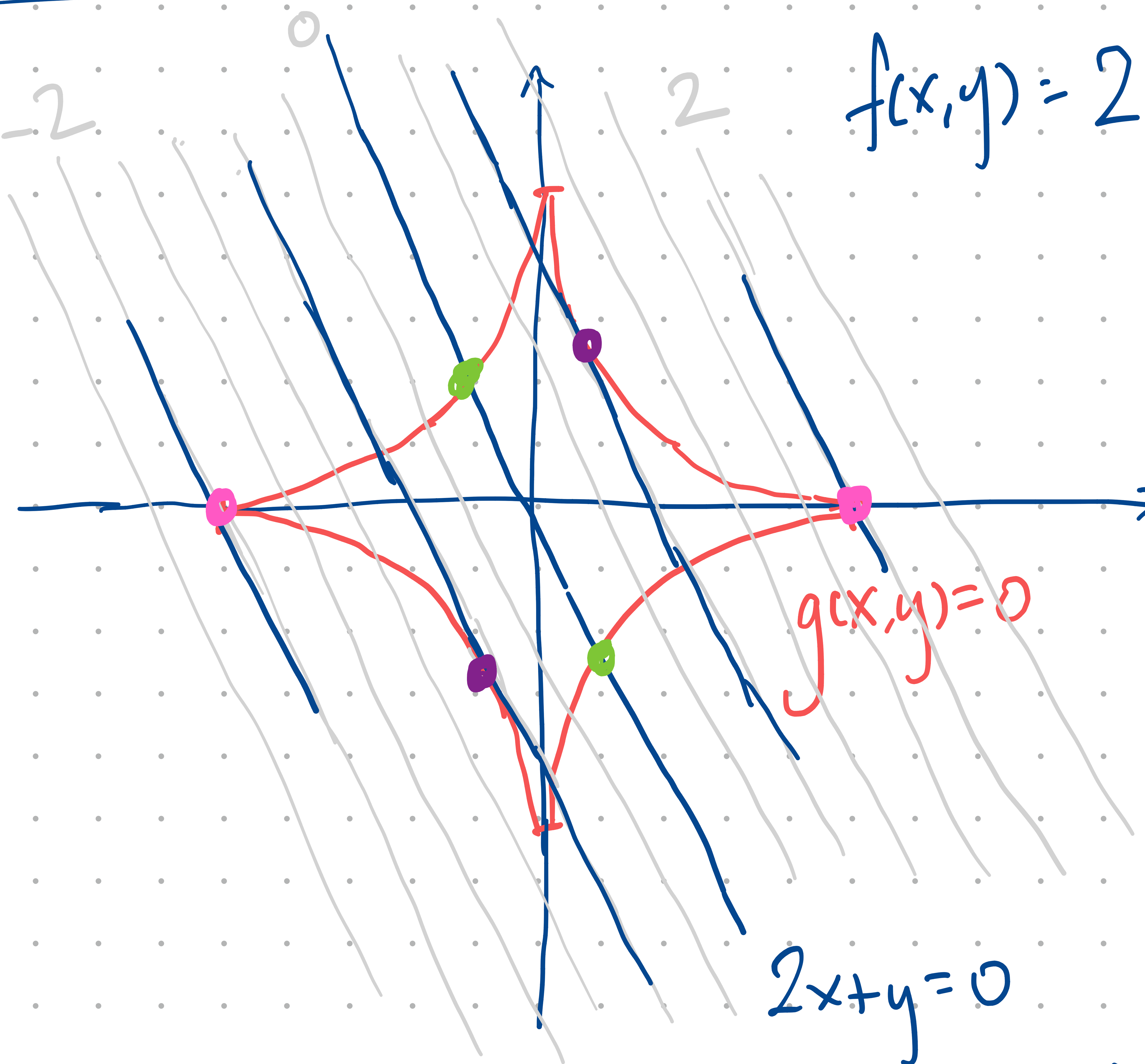
③



$f(x,y) = 2x+y$

$2x+y=0$

i.e. 0 - level set of f

- These 2 points are just points on constraint curve where $f(x,y)=0$, <u>not</u> extrema candidates.

- These 2 points are where $\nabla f = \lambda \nabla g$ for some λ.
(a)

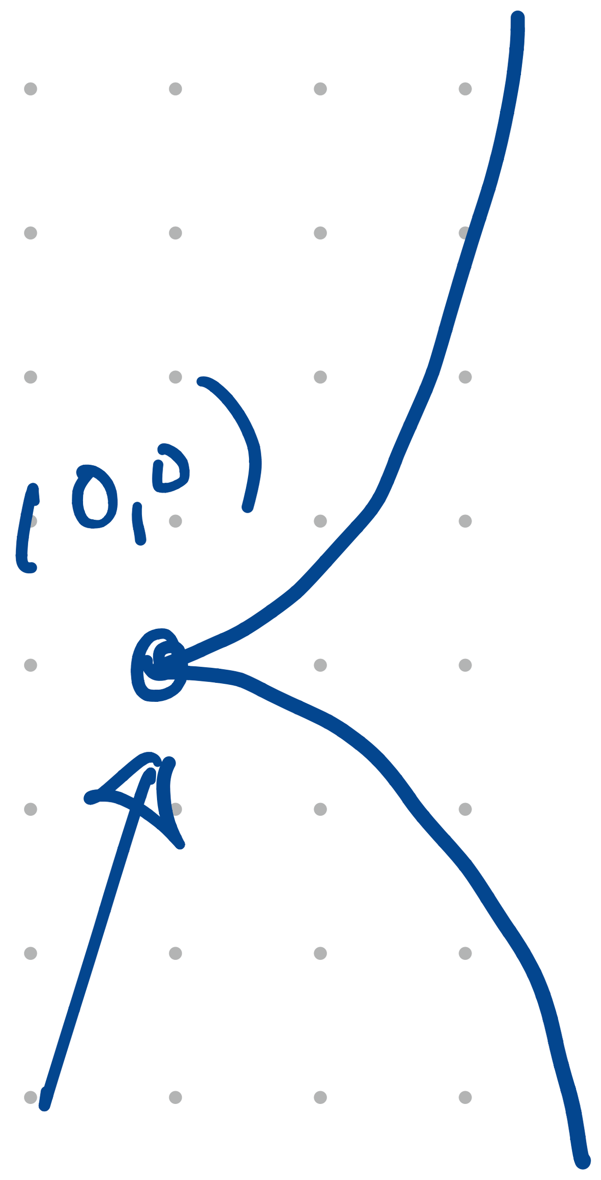
(b) Max @ $f(1,0)=2$

Min @ $f(-1,0)=-2$

(c) Lagrange multi can break @ "singular" points of the constraint curve

"not smooth"

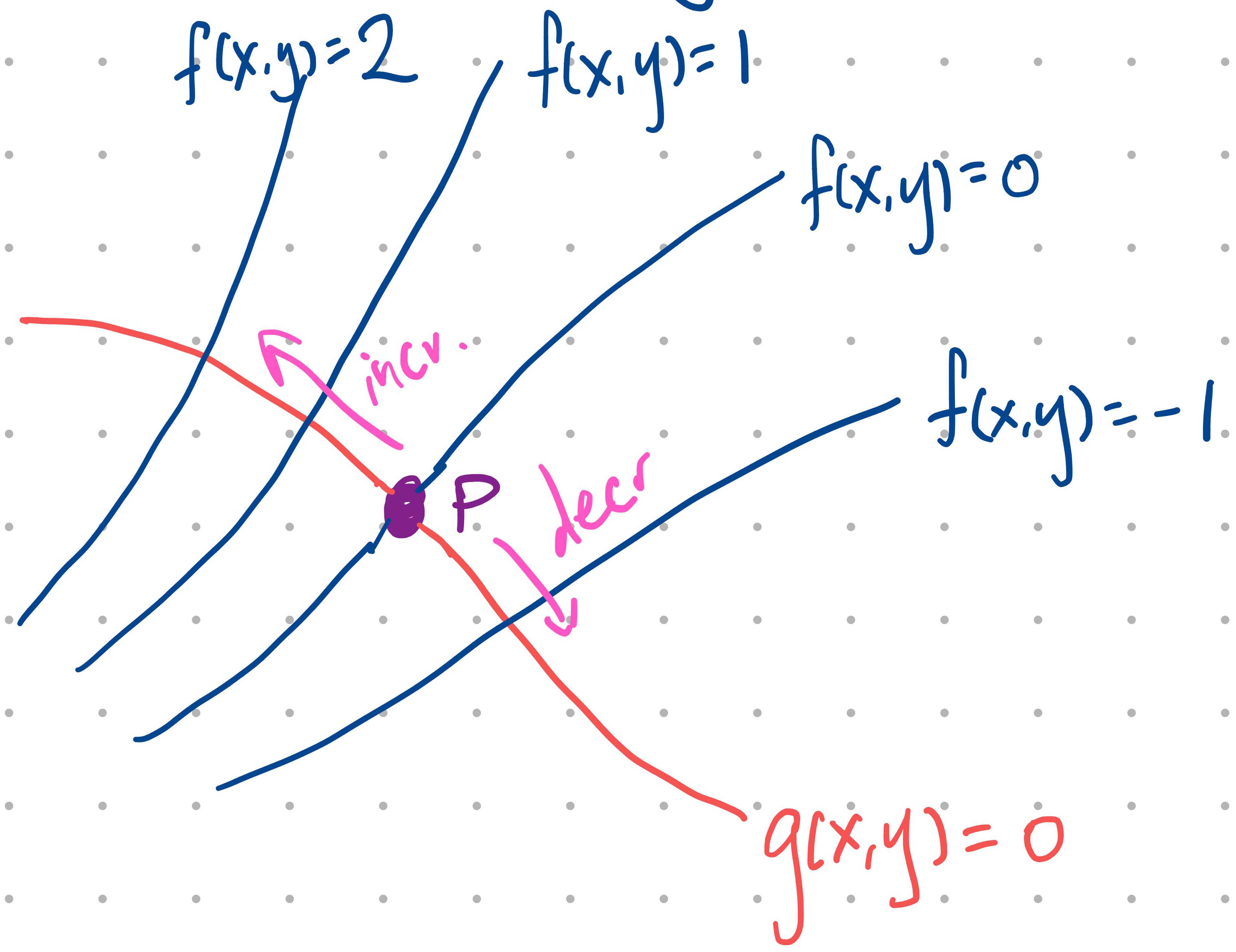
i.e. ∇g is $\vec{0}$ or DNE

e.g. 

$\underbrace{y^2-x^3}_{g(x,y)}=0$

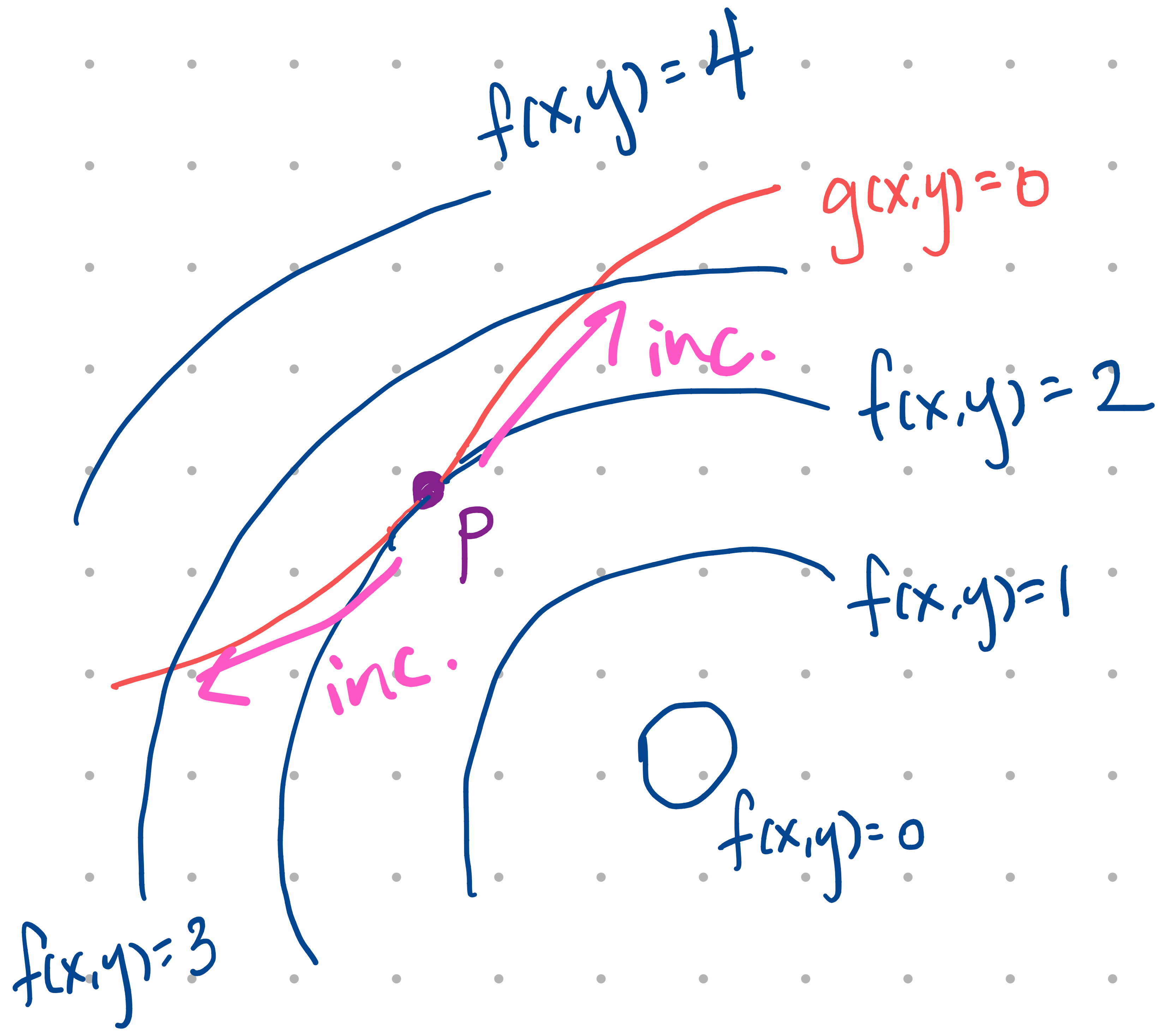
$\nabla g(0,0)=\langle -3(0)^2, 2(0)\rangle=\langle 0,0\rangle$

Assume f and g are differentiable in all these examples.



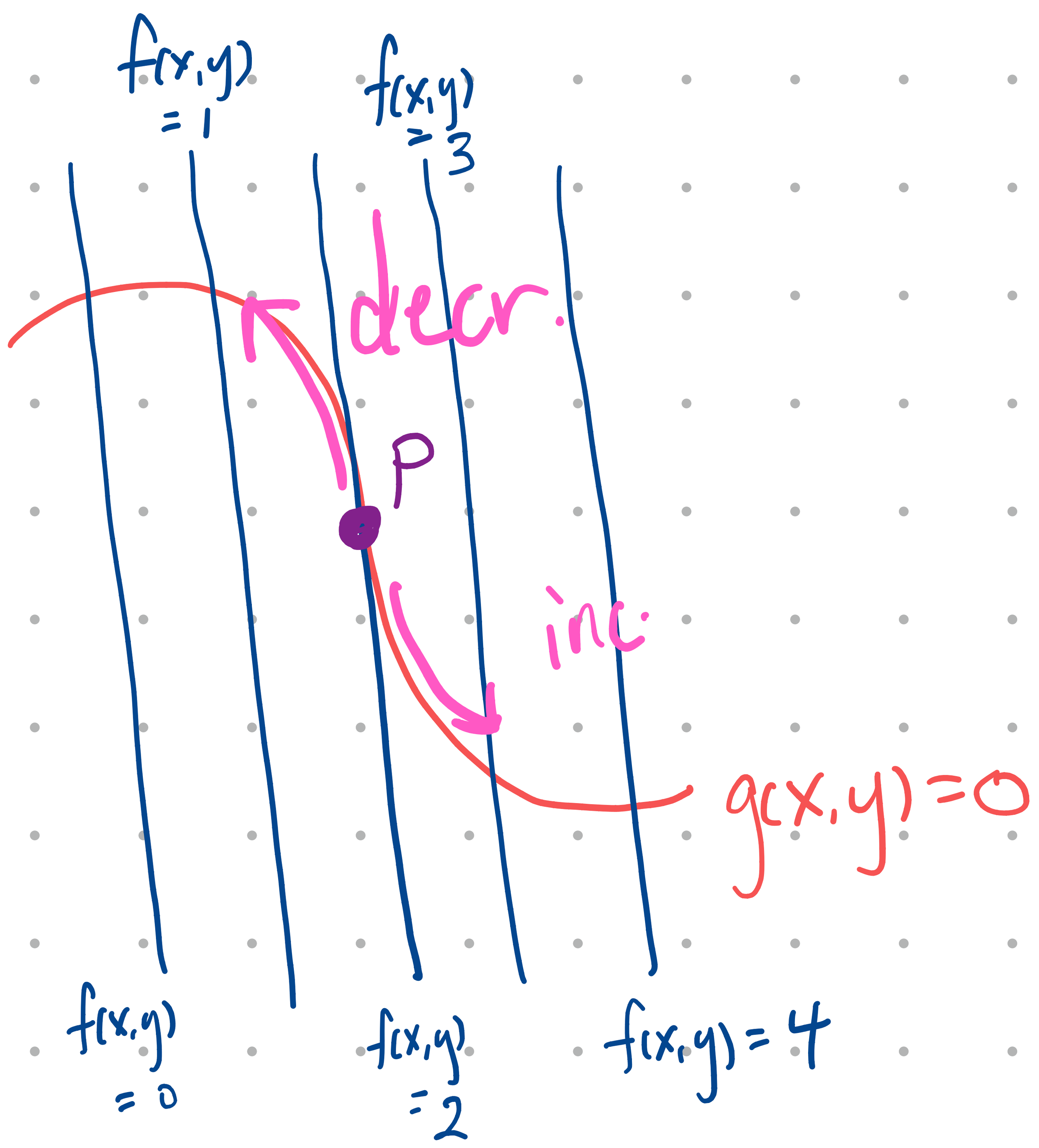
P is neither local max nor min

$$\nabla f(P) \neq \lambda \nabla g(P)$$



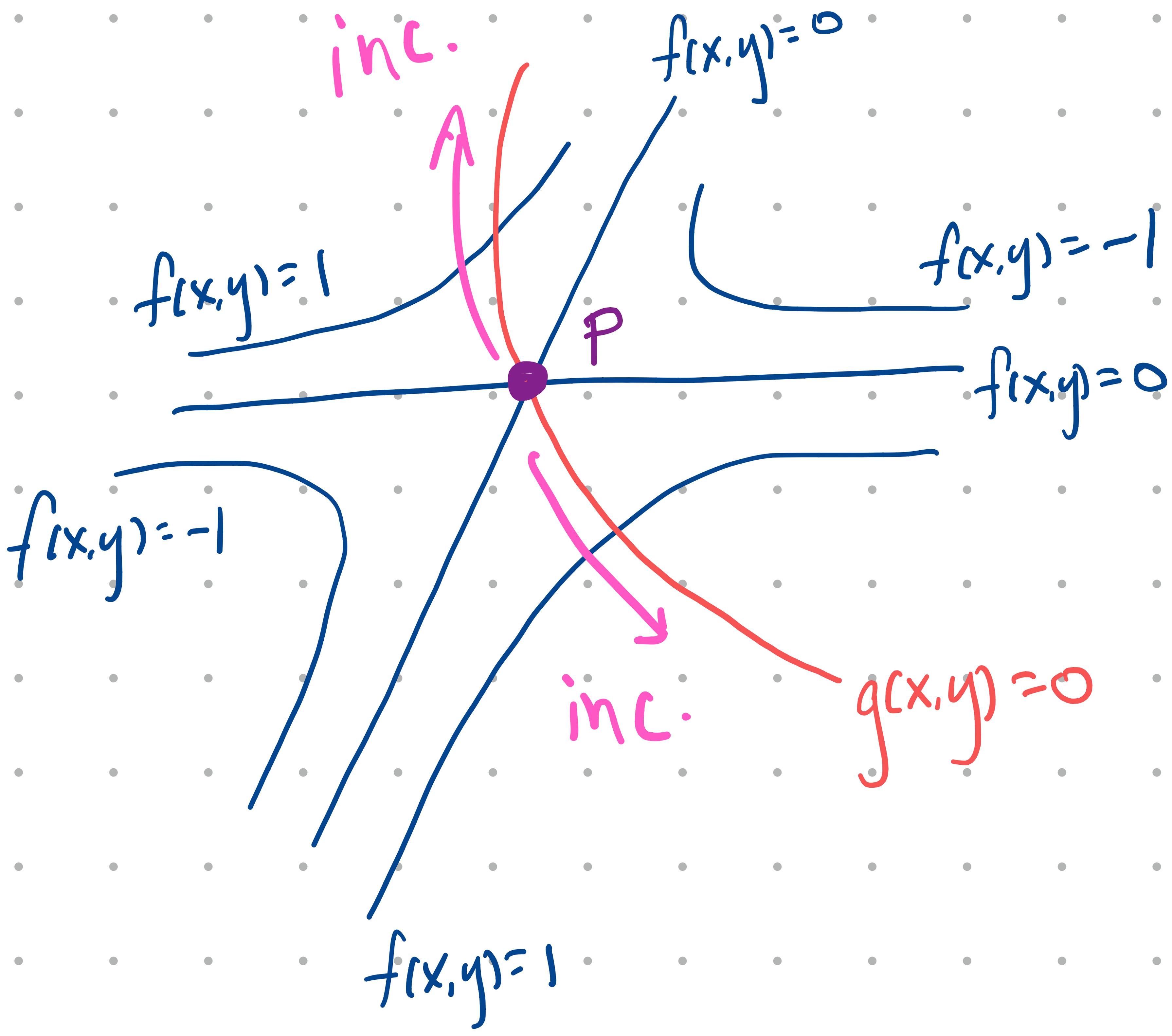
P is local min

$$\nabla f(P) = \lambda \nabla g(P)$$



P is neither local max nor min

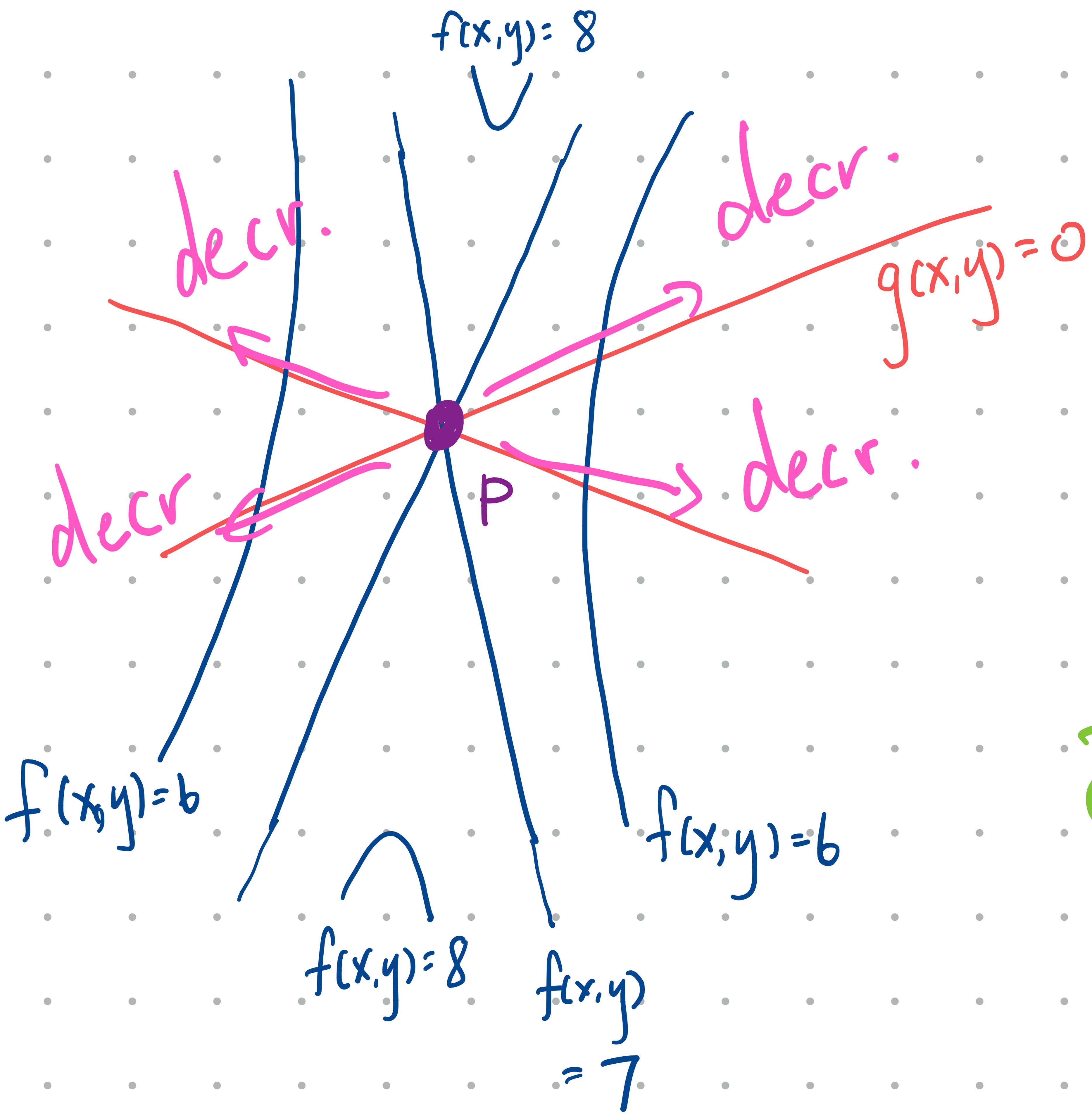
$$\nabla f(P) = \lambda \nabla g(P)$$



P is local min

$$\nabla f(P) = \lambda \nabla g(P)$$

$\nabla f(P) = \vec{0}$ if $\lambda = 0$.

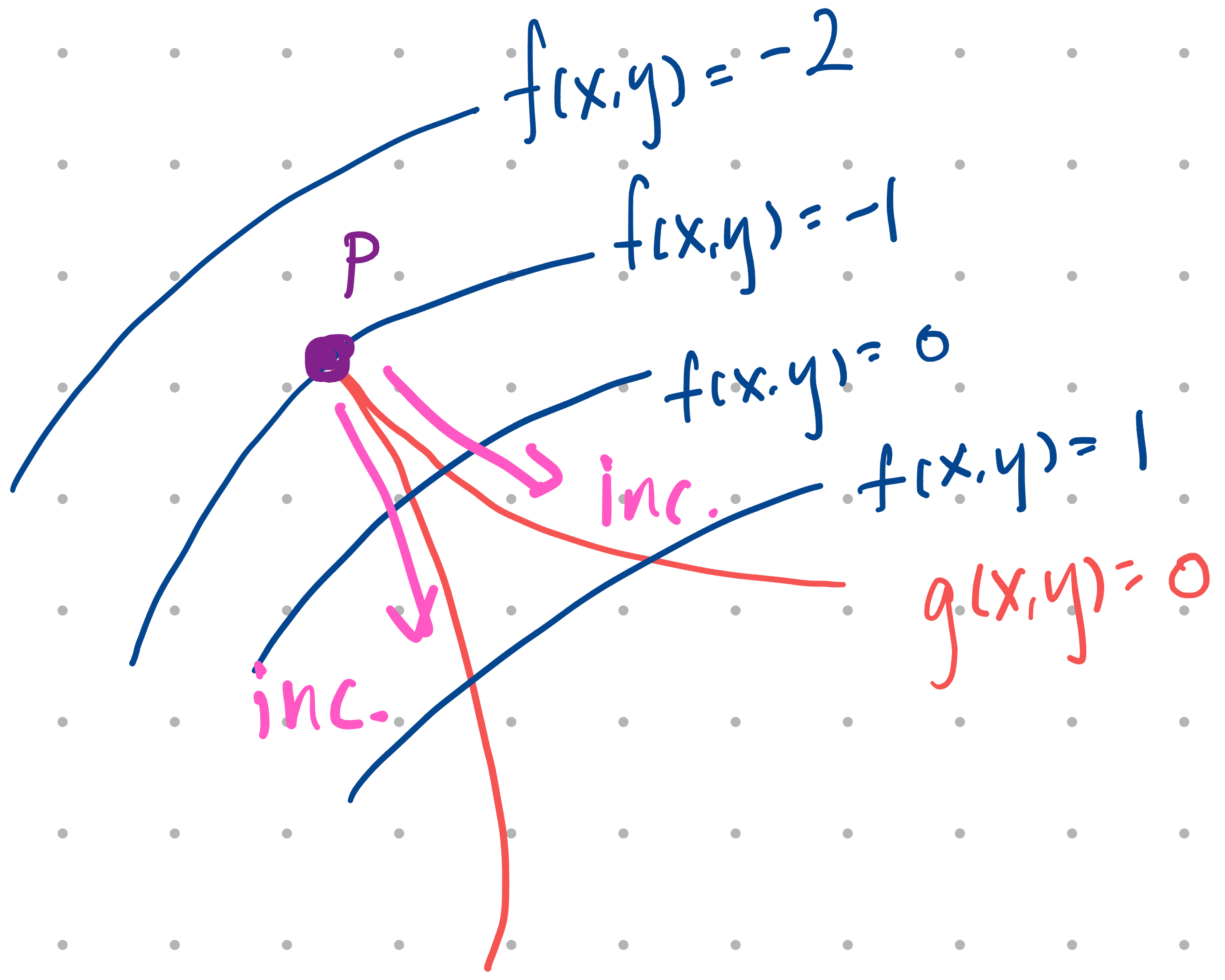


P is local max

$$\nabla f(P) = \lambda \nabla g(P)$$

$\nabla f(P) = \vec{0}$, $\nabla g(P) = \vec{0}$

actually true for any λ....



P is local min

$$\nabla f(P) \neq \lambda \nabla g(P) = \vec{0}$$

nonzero